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RING TOOL PROFILING TO GENERATE A HELICAL SURFACE, USING THE VIRTUAL CONTACT POINT METHOD

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Abstract: Helical surfaces with cylindrical form and constant pitch can be generated using tools that are delimited by primary peripheral revolution surfaces. This tool's profiling was analyzed using fundamental theoretical methods, focusing on the concept of generating the geometry through enveloping involving a revolution surface, which defines the tool's primary peripheral geometry. The virtual contact points theorem extends the general concept of enwrapping surfaces, offering a distinct perspective that identifies the contact points between the ring tool's outer contour and the constant-pitch cylindrical helical surface. Therefore, an application of this method was made in order to obtain the ring tool profile, which will generate a helical surface of cylindrical shape and constant pitch, having a rectilinear profile in the axial plane. Implemented in MATLAB, the application generated results that demonstrate the reliability and the practical simplicity of using the virtual contact point theorem.
Key words: virtual point method, ring tool profiling, curl threading, enwrapping surfaces.

1. INTRODUCTION

The generation of helical surfaces of cylindrical shape and constant pitch can be obtained by means of tools that are delimited by primary peripheral revolution surfaces, such as side mill tools, end mill tools, or ring tools. Ring tools are commonly used in the machining of long helical surfaces, particularly those with relatively small pitch values. Their design, based on peripheral primary revolution surfaces, provides a key advantage: the cutting process achieves a high level of productivity due to the tool's geometry and continuous contact with the part [1, 2].

The tools' profiling was analyzed using fundamental theoretical methods, focusing on the concept of generating the geometry through enveloping involving a revolution surface, which defines the tool's primary peripheral geometry. This surface defines the theoretical geometry on which the tools of the teeth's cutting edges are located and serves as a reference from which the tool's shape is started during manufacturing [3].

The enveloping between the two surfaces, that of the part to be obtained and that of the generating tool, involves a linear contact and can be analyzed using the first Olivier theorem [4]. The problem can also be approached by using Gohman's kinematic theorem of enveloping surfaces, applicable to reciprocally enwrapping surfaces [5, 6]. Such surfaces share a contact curve along which their normals are aligned. This curve is referred to as the characteristic curve [7].

At "Dunărea de Jos" University, several complementary theorems have been formulated over time to address the problem of linear contact between reciprocally enveloping surfaces, including the minimum distance theorem, the in-plane trajectories theorem, and the theorem of the family of substitute circles, among others [8-10]. All these complementary theorems establish a mathematically rigorous basis for generating tool profiling, particularly by defining their primary peripheral geometry - expressed as a revolution surface - on which the tool's active cutting contours are built. This paper presents an additional approach to ring tool profiling, called the "virtual contact point" theorem, which applies to tools used in the processing of reciprocally enwrapping surfaces with linear contact. Originally elaborated for planar enveloping profiles defined by a set of rolling centrodes, the method was subsequently generalized to accommodate surface geometries characterized by linear

contact enveloping conditions [11].
The virtual contact points theorem extends the general concept of enwrapping surfaces, established by Olivier and Gohman, offering a distinct perspective that identifies the contact points between the ring tool's outer contour and the constant-pitch cylindrical helical surface [12]. Therefore, an application of this method was made in order to obtain the ring tool profile, which will generate a helical surface of cylindrical shape and constant pitch, having a rectilinear profile in the axial plane. Implemented in MATLAB, the application generated results that demonstrate the reliability and the practical simplicity of using the virtual contact point theorem. Previous research has shown that this theorem can be applied not only to the profiling of different types of tools but also to the profiling of various surface geometries.

2. VIRTUAL CONTACT POINT METHOD

The theorem has been used in the profiling of disk-type tools, especially for the generation of surfaces of cylindrical helical shape with constant pitch and circular cross-sections in the frontal plane [12].
As established, when the planar generator of a constant-pitch cylindrical helical surface can be defined through parametric equations [3], [5, 6]:

$$G:\begin{cases} x = x(u); \\ y = y(u); \\ z = 0, \end{cases} \tag{1}$$

the helical surface equation will be given by:

$$x(u,\varphi) = \omega_3^T(\varphi)\cdot x(u) + p_e\cdot\varphi\cdot\vec{k}\ , \tag{2}$$

where *x(u)* is the matrix formed with the coordinates of *x*, *y,* and *z* of a point which belongs to $x(u)=\begin{pmatrix} x(u) \\ y(u) \\ z(u) \end{pmatrix}$ generator; $\omega_3^T(\varphi)=\begin{pmatrix} cos\varphi & -sin\varphi & 0 \\ sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$ - the matrix that rotates around *z*; p_e - the parameter defining the helical movement; $p_e = \dfrac{p}{2\cdot\pi}$, *p* - the pitch of the helix [3].

In (2) the surface Σ is generated by the following equations [3]:

$$\Sigma:\begin{cases} x(u,\varphi) = x(u)\cdot cos\varphi - y(u)\cdot sin\varphi; \\ y(u,\varphi) = x(u)\cdot sin\varphi + y(u)\cdot cos\varphi; \\ z(u,\varphi) = p_e\cdot\varphi. \end{cases} \tag{3}$$

Assuming that the Σ surface is produced by a tool whose main peripheral surface is a surface of revolution, *S*, the interaction between Σ and S takes place along a curve denoted as $C_{\Sigma S}$, referred to as the characteristic curve [3]. Since each point on the helical surface eventually lies on this characteristic curve, it consequently belongs to the generated surface *S*.
Thus, each point on the intermediate surface Σ can be interpreted as a "virtual contact point" with the enwrapping surface. To identify the characteristic curve, it is necessary to establish the enveloping condition, which determines the relationship between the independent parameters *u* and φ from equation (2), when the point is part of the characteristic curve. Within the virtual contact point theorem, the Gohman method is adopted as the enveloping condition. Under this condition, at every point along the characteristic curve, the vector of normal to the intermediate surface Σ is orthogonal to the vector of velocity produced by the tool's rotational motion around its axis [12]:

$$\vec{N}_{\Sigma} \perp \vec{v} \Leftrightarrow \vec{N}_{\Sigma} \cdot \vec{v} = 0 . \tag{4}$$

Assuming that the tool is defined in its own local coordinate system, denoted by *XYZ*, the axis $\vec{A}$ (being the ring tool axis) can be expressed as follows:

$$\vec{A} = A_X \cdot \vec{i} + A_Y \cdot \vec{j} + A_Z \cdot \vec{k} , \tag{5}$$

where A_X, A_Y, and A_Z represent the projections of the tool's axis onto the *X*, *Y*, and *Z* axes of the system.
Figure 1 shows the relative positioning of the two coordinate systems: the one associated with the helical surface (*xyz*) and the one associated with the ring tool (*XYZ*).

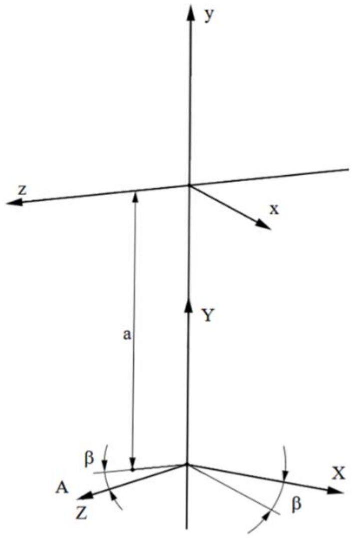

Fig. 1. Orientation of the part (*xyz*) and tool (*XYZ*) coordinate systems [3].

If it is accepted that the tool reference system has been chosen so that the *Z* axis coincides with the axis of the tool and the *Y* axis overlaps the *y* axis, the tool axis projections onto the axes of the own reference system are given by the equations:

$$\vec{A} : \begin{cases} A_X = 0; \\ A_Y = 0; \\ A_Z = t, \end{cases} \tag{6}$$

t being a scalar parameter, variable along the $\vec{A}$ axis.
The coordinate transformation between the *xyz* and *XYZ* systems is given by:

$$x = \omega_2^T(\beta) + B , \tag{7}$$

where β represents the inclination angle of the tool system axes relative to the system of the part, the angle of a helical line situated on the Σ surface, measured relative to the axis of that surface:

$$\beta = arctan\left(\frac{p}{R}\right), \tag{8}$$

and $B = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$, see also Figure 1. Also, the *R* parameter from equation (8) is the radius of the circle on which the helix is defined. The *a* value is chosen from constructive considerations of the tool, to ensure a sufficiently large radius for it.
Therefore, the orientation of the tool axis within the coordinate system, which is associated with the part, is defined as follows:

$$\vec{A}_x = (t \cdot sin\,\beta) \cdot \vec{i} - a \cdot \vec{j} + (t \cdot cos\,\beta) \cdot \vec{k}\,. \tag{9}$$

The normal vector to the surface Σ is determined using the following determinant:

$$\vec{N}_\Sigma = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dot{x}_u & \dot{y}_u & \dot{z}_u \\ \dot{x}_\varphi & \dot{y}_\varphi & \dot{z}_\varphi \end{vmatrix}, \tag{10}$$

where $\dot{x}_u, \dot{y}_u, \dot{z}_u$ and $\dot{x}_\varphi, \dot{y}_\varphi, \dot{z}_\varphi$ represent the partial derivatives of the Σ surface equations relative to an independent parameter u, respectively φ.

To follow the enveloping condition, the normal vector to the helical surface Σ will intersect the ring tool axis at the current point M. This geometric constraint allows the determination of contact points and the construction of a characteristic curve.
Assuming that the position vector of the point N, which is positioned along the normal to the surface Σ, can be expressed in the form:

$$\vec{n}_M = (\dot{y}_u \cdot \dot{z}_\varphi - \dot{y}_\varphi \cdot \dot{z}_u) \cdot \lambda \cdot \vec{i} + (\dot{x}_\varphi \cdot \dot{z}_u - \dot{x}_u \cdot \dot{z}_\varphi) \cdot \lambda \cdot \vec{j} + (\dot{x}_u \cdot \dot{y}_\varphi - \dot{x}_\varphi \cdot \dot{y}_u) \cdot \lambda \cdot \vec{k}\,, \tag{11}$$

which specifies the distance from the current point M measured along the normal direction, and assuming that point N is on the tool axis - thus imposing the intersection of the axis - the following equations system can be formulated:

$$\begin{cases} t \cdot sin\,\beta = (\dot{y}_u \cdot \dot{z}_\varphi - \dot{y}_\varphi \cdot \dot{z}_u) \cdot \lambda; \\ -a = (\dot{x}_\varphi \cdot \dot{z}_u - \dot{x}_u \cdot \dot{z}_\varphi) \cdot \lambda; \\ t \cdot cos\,\beta = (\dot{x}_u \cdot \dot{y}_\varphi - \dot{x}_\varphi \cdot \dot{y}_u) \cdot \lambda. \end{cases} \tag{12}$$

From the equations system, the λ parameter can be eliminated, obtaining the enveloping condition in the form:

$$\varphi = \varphi(u). \tag{13}$$

3. RING TOOL FOR PROCESSING THE SCREWED SHAFT

Screwed shafts are cylindrical parts on the surface of which two helical channels in opposite directions (left and right) are made to allow the automatic change of the longitudinal direction of the cable being wound onto or off the drum. Usually, these channels are processed with end mills, but this implies a relatively low productivity.
By processing with ring tools, productivity is greatly increased.
The axial section of one of the processed channels is presented in Figure 2. The generator of a helical surface, denoted by G, is described by the equations:

$$G: \begin{cases} x = 0; \\ y = u \cdot cos\,\alpha; \\ z = u \cdot sin\,\alpha. \end{cases} \tag{14}$$

The helical movement is described by:

$$x = \omega_3(\varphi) \cdot x\,, \tag{15}$$

or, in a developed form:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} cos\,\varphi & sin\,\varphi & 0 \\ -sin\,\varphi & cos\,\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ u \cdot cos\,\alpha \\ u \cdot sin\,\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p_e \cdot \varphi \end{pmatrix} = \begin{pmatrix} u \cdot cos\,\alpha \cdot sin\,\varphi \\ u \cdot cos\,\alpha \cdot cos\,\varphi \\ u \cdot sin\,\alpha + p_e \cdot \varphi \end{pmatrix} \tag{16}$$

Therefore, the helical surface is represented by the following equations:

$$\Sigma:\begin{cases}x=u\cdot cos\,\alpha\cdot sin\,\varphi;\\ y=u\cdot cos\,\alpha\cdot cos\,\varphi;\\ z=u\cdot sin\,\alpha+p_e\cdot\varphi.\end{cases}\tag{17}$$

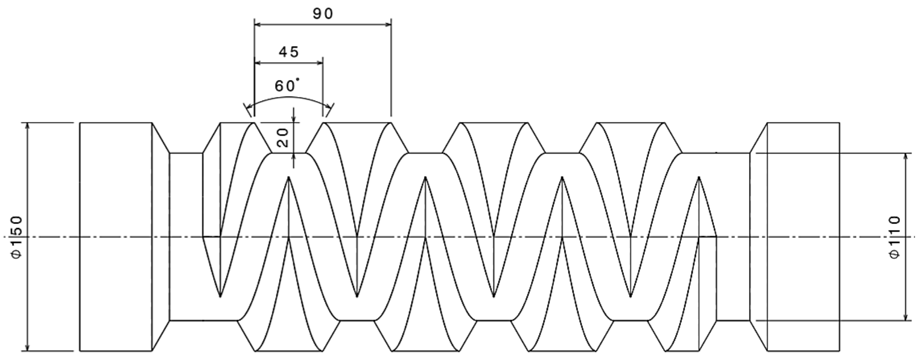

Fig. 2. The constructive dimensions of the "screwed shaft" part

Under these conditions, the profile's normal vector will have the direction:

$$\vec{n}_\Sigma=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ cos\,\alpha\cdot sin\,\varphi & cos\,\alpha\cdot cos\,\varphi & sin\,\alpha\\ u\cdot cos\,\alpha\cdot cos\,\varphi & -u\cdot cos\,\alpha\cdot sin\,\varphi & p_e\end{vmatrix}=$$

$$=\left(p_e\cdot cos\,\varphi+u\cdot sin\,\alpha\cdot sin\,\varphi\right)\cdot cos\,\alpha\cdot\vec{i}-\left(p_e\cdot sin\,\varphi-u\cdot sin\,\alpha\cdot cos\,\varphi\right)\cdot cos\,\alpha\cdot\vec{j}-\tag{18}$$
$$-u\cdot cos^2\,\alpha\cdot\vec{k}.$$

Therefore, the normal vector generated from the current point on Σ surface toward the axis of the ring tool is expressed as:

$$\begin{aligned}\vec{n}_\Sigma&=\left(p_e\cdot cos\,\varphi+u\cdot sin\,\alpha\cdot sin\,\varphi\right)\cdot cos\,\alpha\cdot\lambda\cdot\vec{i}-\\&-\left(p_e\cdot sin\,\varphi-u\cdot sin\,\alpha\cdot cos\,\varphi\right)\cdot cos\,\alpha\cdot\lambda\cdot\vec{j}-u\cdot cos^2\,\alpha\cdot\lambda\cdot\vec{k},\end{aligned}\tag{19}$$

where λ is the scalar parameter that defines the distance measured from the selected point on surface Σ to the point of intersection with the tool axis.

On the other hand, the tool axis vector, in the *XYZ* system, has the form:

$$\vec{A}:\begin{cases}X=0;\\ Y=0;\\ Z=t,\end{cases}\tag{20}$$

which, in the *xyz* system, becomes:

$$x=\omega_2^T\left(\beta\right)\cdot X+A=\begin{pmatrix}cos\,\beta & 0 & sin\,\beta\\ 0 & 1 & 0\\ -sin\,\beta & 0 & cos\,\beta\end{pmatrix}\cdot\begin{pmatrix}0\\0\\t\end{pmatrix}+\begin{pmatrix}0\\-a\\0\end{pmatrix}=\begin{pmatrix}t\cdot sin\,\beta\\-a\\t\cdot cos\,\beta\end{pmatrix}\tag{21}$$

Identifying the enveloping condition involves solving the system of equations:

$$\begin{cases}t\cdot sin\,\beta=\left(p_e\cdot cos\,\varphi+u\cdot sin\,\alpha\cdot sin\,\varphi\right)\cdot cos\,\alpha\cdot\lambda;\\ -a=-\left(p_e\cdot sin\,\varphi-u\cdot sin\,\alpha\cdot cos\,\varphi\right)\cdot cos\,\alpha\cdot\lambda;\\ t\cdot cos\,\beta=-u\cdot cos^2\,\alpha\cdot\lambda.\end{cases}\tag{22}$$

The system (22) enables the λ and t parameters:

$$\lambda = \frac{a}{\left(p_e \cdot sin\varphi - u \cdot sin\alpha \cdot cos\varphi\right) \cdot cos\alpha}, \tag{23}$$

$$\frac{t \cdot sin\beta}{t \cdot cos\beta} = \frac{\left(p_e + u \cdot sin\alpha \cdot sin\varphi\right) \cdot cos\alpha}{-u \cdot cos^2\alpha}, \tag{24}$$

$$tg\beta = \frac{\left(p_e\, cos\varphi + u \cdot sin\alpha \cdot sin\varphi\right) \cdot cos\alpha}{-u \cdot cos^2\alpha}, \tag{25}$$

and, through successive calculations, it will reach at the form:

$$u = \frac{-p_e \cdot cos\varphi}{\left(sin\alpha \cdot sin\varphi + cos\alpha \cdot tg\beta\right)}. \tag{26}$$

The characteristic curve is obtained by combining equations (17) and (26) and within the coordinate systems which are associated with the part. The transformation to the *XYZ* system, which is associated with the tool, is performed through the following transformation:

$$X = \omega_2\left(\beta\right) \cdot x + A,\ A = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}. \tag{27}$$

In order to find the results that satisfy the enveloping condition, a dedicated script was created in MATLAB calculation program, using the *MATLAB vpasolve* function, with which the numerical solution of a symbolic equation can be found.
The application of this computational approach offers significant advantages compared to classical methods, particularly when dealing with complex formulations of the enveloping condition.
The helical surface *Σ*, the generator *G,* and the characteristic curve *CC* are shown in Figure 3. The points determined along the characteristic curve are given in Table 1, and the points corresponding to the ring tool generator are centralized in Table 2.
The coordinates from Table 1 are calculated along the characteristic curve, both in the part and tool coordinate systems. These points mark the successive positions where the virtual contact between the helical surface and the tool revolution surface is achieved.
The values in Table 2 are the result of transforming these points into the axial plane of the tool, describing the change in the profile radius as a function of the axial height. Thus, the profile shown in Figure 4 is built based on these coordinates and represents the active contour of the tool, necessary for generating the helical surface with constant pitch.

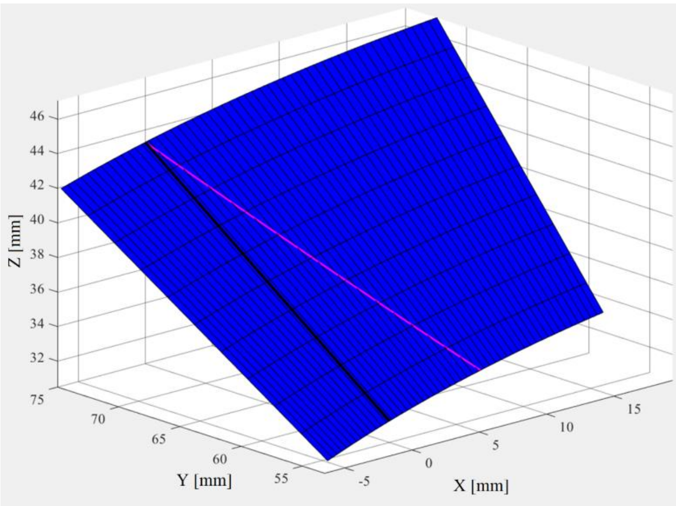

Fig. 3. The generator, surface, and characteristic curve

Table 1. The coordinates of the points identified along the characteristic curve

In the part's coordinate system			*In the tool's coordinate system*		
x [mm]	*y [mm]*	*z [mm]*	*X [mm]*	*Y [mm]*	*Z [mm]*
6.445	54.621	33.437	0.058	304.621	34.052
5.824	56.702	34.375	-0.728	306.702	34.858
5.196	58.771	35.327	-1.523	308.771	35.674
4.562	60.829	36.291	-2.327	310.829	36.502
3.921	62.878	37.265	-3.139	312.878	37.339
3.276	64.917	38.250	-3.957	314.917	38.186
2.627	66.948	39.244	-4.781	316.948	39.040
1.975	68.972	40.247	-5.611	318.972	39.903
1.319	70.988	41.258	-6.444	320.988	40.773
0.661	72.997	42.276	-7.282	322.997	41.650
0.000	75.000	43.301	-8.123	325.000	42.533

Table 2. Coordinates of the points on the tool's axial section

H [mm]	*R [mm]*
54.621	34.052
56.702	34.865
58.771	35.707
60.829	36.576
62.878	37.471
64.917	38.39
66.948	39.332
68.972	40.296
70.988	41.279
72.997	42.281
75.000	43.301

Figure 4 shows the ring tool's axial section. The shape of the profile in this section results from determining the points where the normal of the helical surface intersects the tool axis, respecting the enwrapping condition imposed by the method used.

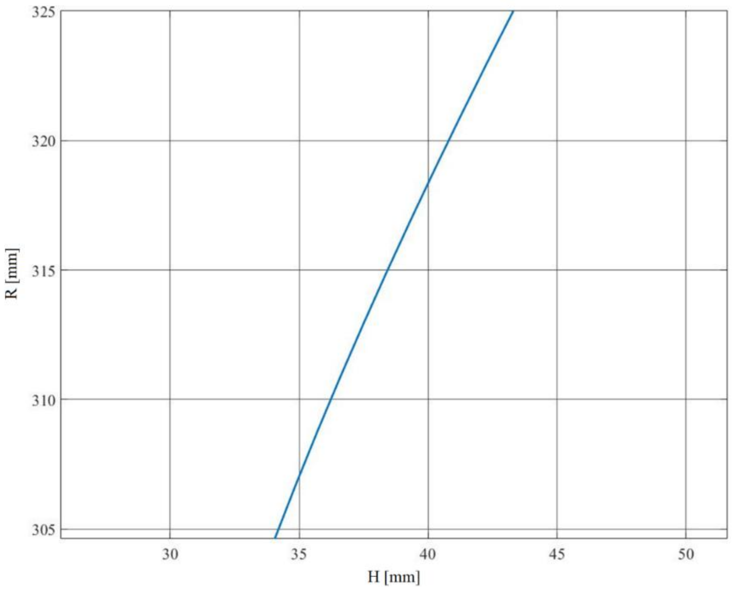

Fig. 4. *SA* - axial section

4. CONCLUSIONS

In this paper, the virtual contact point method was applied to the geometrical profiling of a cylindrical helical surface of constant pitch. The generator of the surface is a planar curve, specifically a linear segment; however, the methodology is sufficiently versatile to handle generating curves described by more complicated mathematical equations.

Furthermore, the method has been extended to the profiling of an alternative tool type - the disk tool, which is used in generating cylindrical helical surfaces of constant pitch, in cases where the generator is defined within the frontal plane.
The results show the quality of the theorem and the possibility of extending its applicability from the study of plane enwrapping problems to spatial problems, such as the enwrapping between helical surfaces with revolution surfaces. The originality of the method consists of using the MATLAB program's ability to solve symbolic calculation problems, which reduces the risk of error and ensures the flexibility of the designed programs.
Practically, for a certain type of tool, a single program can meet the calculation needs because only the form of the generating curve and the enveloping condition change.

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